

**CONSTANT AREA MIXING
OF NON-ISOENERGETIC
COAXIAL COMPRESSIBLE STREAMS**

By

**C.E. Peters and S. Wehofer
Rocket Test Facility
ARO, Inc.**

January 1962

**ARNOLD ENGINEERING DEVELOPMENT CENTER
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE**

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ABSTRACT

As part of a general investigation of two-stream supersonic diffusers, a theory was developed for the bounded turbulent mixing between coaxial high velocity streams of different composition and temperature. The usual assumption of negligible axial pressure gradient is invalid in this case because the outer stream is bounded by a cylindrical duct. A highly simplified flow model is used which retains the essential features of the actual mixing process; the resulting system of equations is amenable to manual calculation.

The theory is correlated with data obtained from an annular nozzle wind tunnel having a central supersonic core of rocket exhaust gases. Correlation is also made with Mikhail's low velocity data for which his more elaborate theory is available for comparison. The simplified theory predicts with reasonable accuracy the experimental axial static pressure and velocity distributions.

CONTENTS

| | <u>Page</u> |
|--|-------------|
| ABSTRACT | 5 |
| NOMENCLATURE | 7 |
| INTRODUCTION | 9 |
| DEVELOPMENT OF THEORY | 9 |
| EXPERIMENTAL INVESTIGATION | 17 |
| DISCUSSION OF RESULTS | 18 |
| CONCLUSIONS | 21 |
| REFERENCES | 21 |
| APPENDIX - Iteration Procedure for Mixing Analysis | 23 |

TABLE

| | |
|--|----|
| Tunnel Dimensions and Operating Parameters | 25 |
|--|----|

ILLUSTRATIONS

Figure

| | |
|---|----|
| 1. Schematic of Tunnel for Two-Stream Diffuser Investigation | 27 |
| 2. Schematic of Velocity Profiles for Coaxial Stream Duct Flow | 28 |
| 3. Approximation of Actual Velocity Profiles for Coaxial Duct Flow | 29 |
| 4. Mixing Model for Calculation of Local Rates of Heat and Momentum Transfer | 30 |
| 5. Dividing Streamline Velocity Ratio | 31 |
| 6. Mach Number Function for Air and Typical Rocket Products | 32 |
| 7. Variation of Mixing Zone Spreading Rate with Mach Number for Zero Secondary Velocity | 33 |
| 8. Variation of Turbulent Mixing Parameter with Mach Number for Fully Developed Mixing of an Axisymmetric Jet | 34 |

| <u>Figure</u> | <u>Page</u> |
|--|-------------|
| 9. Schematic of Mikhail's Experimental Apparatus | 35 |
| 10. Comparison of Theory with Mikhail's Experimental Data | 36 |
| 11. Comparison of Theoretical and Experimental Centerline Velocity Distribution | 37 |
| 12. Schematic of Supersonic Jet Expanding into Supersonic Stream | 38 |
| 13. Axial Static Pressure Distribution for Mach Number 3.07 Tunnel at $T_{oa} = 1100^{\circ}R$ | |
| a. $P_{oa} = 40.8$ psia | 39 |
| b. $P_{oa} = 23.5$ psia | 40 |
| c. $P_{oa} = 16.0$ psia | 41 |
| 14. Theoretical Pressure and Mach Number Distributions for Mach Number 3.07 Tunnel | 42 |
| 15. Theoretical Outer Stream Mach Number Distribution for Mach Number 3.07 Tunnel | 43 |

NOMENCLATURE

| | |
|------------|---|
| A | Cross-sectional area |
| b | Width of mixing zone |
| c_{fw} | Skin friction coefficient at wall |
| c_p | Specific heat |
| D | Diameter |
| F | Total stream thrust |
| J | Joule's constant, mechanical equivalent of heat |
| k | Turbulent mixing parameter |
| M | Mach number |
| p | Static pressure |
| P_o | Total pressure |
| \dot{Q} | Total energy transfer rate |
| q | Local energy flux between streams |
| R | Gas constant |
| T_o | Total temperature |
| u | Axial velocity |
| w | Weight flow |
| x | Axial distance from initiation of mixing |
| y | Coordinate in mixing model |
| γ | Ratio of specific heats |
| ϵ | Apparent kinematic viscosity |
| ρ | Density |
| σ | Turbulent mixing parameter |
| τ | Shear stress |

SUBSCRIPTS

| | |
|------|-------------------------|
| 1, 2 | Refers to cross section |
| a | Outer stream conditions |

| | |
|-----|--|
| avg | Arithmetic average conditions over an interval |
| c | Conditions along dividing streamline |
| D | Duct |
| j | Conditions in core flow |
| max | Maximum velocity adjacent to mixing region |
| min | Minimum velocity adjacent to mixing region |
| w | Conditions at duct wall |

INTRODUCTION

An experimental and theoretical investigation of supersonic two-stream diffuser operation is currently in progress at the Rocket Test Facility (RTF), Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC). The investigation is concerned with operation of an axisymmetric wind tunnel having a central core of rocket exhaust gases. Such a diffuser is normally operated with a fully developed duct shock system which causes rapid mixing of the streams. Overall diffuser performance may be accurately predicted for such a case by using one-dimensional theory.

At certain operating conditions, however, it is desirable to maintain the back-pressure-induced shock system far downstream in the diffuser, and for this case the mixing characteristics of the two streams while both streams remain supersonic become important. Simple one-dimensional theory is inadequate for analysis of the supersonic mixing process, and since rigorous analysis of the bounded two-stream mixing problem is very difficult, a simplified theory for bounded mixing of coaxial compressible streams has been developed and is presented in this report. A later report will be presented on the general two-stream diffuser investigation. The theoretical and experimental results on two-stream mixing are also applicable to other coaxial stream mixing problems - for example, problems concerning the operation of jet pumps and propulsion system thrust augmentation devices.

The simplified theory was correlated with experimental data obtained at AEDC using the tunnel configuration shown in Fig. 1 operating at low back pressure and with low velocity two-stream mixing data obtained from other agencies.

DEVELOPMENT OF THEORY

Generally, one of two approaches is taken to solve free turbulent mixing problems. The first approach is to write equations of motion for the flow field and find solutions satisfying the boundary conditions. The second approach is to assume velocity profile shapes and then use integral methods to define the flow field. No purely theoretical explanation for

turbulent processes has yet been devised, and at least one empirical constant, a turbulent mixing parameter, must be used for either approach.

For the problem of interest, the mixing of two streams inside a duct, the usual assumption of constant pressure mixing is invalid. Mikhail (Ref. 1), in his treatment of incompressible mixing of coaxial streams in a duct, is one of the few investigators to consider axial pressure gradients. In his case, in spite of the simplifying condition of incompressible flow, a number of assumptions were necessary to solve the integral equations, and three empirical turbulent coefficients were used to correlate theory with experimental data. In addition to axial pressure gradients, it is necessary to include effects of compressibility, energy transfer between the streams, wall friction, and different stream composition in a realistic treatment of the behavior of a rocket jet exhausting into a bounded coaxial supersonic airstream. Considering Mikhail's difficulties with a much less complex case, it is not surprising that the inclusion of these effects in the more classical approaches to turbulent mixing problems results in equations which, if they can be written, are extremely difficult to solve, even with numerical methods. In view of these difficulties, an approach was taken during the present study, which is a considerable departure from earlier attempts to solve such nonconstant pressure mixing problems.

A schematic of the actual flow model for coaxial stream duct flow is shown in Fig. 2. The axial variation of flow conditions in this duct may be treated approximately by assuming that the mass of each stream remains separated and that each is one-dimensional at any section (Fig. 3). If relations for the shear stress and heat transfer between the streams and at the duct wall may be written in terms of the one-dimensional stream properties, then one-dimensional equations for momentum, continuity, and energy may be written for each stream using the unknowns, T_{oa} , T_{oj} , p , A_j , M_a , and M_j .

The assumptions of the analysis are:

1. Flow is steady.
2. Local rates of heat and momentum transfer between the core and outer flow are the same as for mixing between respectively uniform two-dimensional streams.
3. Each stream is initially one-dimensional and remains one-dimensional with no subsequent mass interchange.
4. Each stream acts as a perfect gas having constant specific heat and composition.

5. Turbulent shear stress over a finite interval, Δx , is the average of the shear stress at the end points of the interval.
6. The turbulent Prandtl number for the mixing process is unity.
7. The velocity, temperature, and composition profiles across the mixing zone are linear (Fig. 4).
8. The dividing streamline separating the mass flow of the respective streams occurs at the midpoint of the mixing zone velocity profile.
9. The turbulent exchange parameter, k , is constant for the entire mixing process.
10. The static pressure is constant across any duct cross section.
11. The flow at the duct wall is turbulent, and the wall friction coefficient, c_{fw} , is constant over the entire duct.

TURBULENT TRANSFER QUANTITIES

The local turbulent exchange in the mixing zone was calculated using the simple mixing model of Fig. 4 for mixing between uniform streams. The shear stress in the turbulent region is

$$\tau = \rho \epsilon \frac{\partial u}{\partial y} \quad (1)$$

where ϵ is the apparent turbulent kinematic viscosity. For turbulent Prandtl number unity, the apparent viscosity for energy transfer is identical with that for momentum transfer, and the total energy flux in the turbulent zone is

$$\dot{Q} = \rho c_p \epsilon \frac{\partial T_o}{\partial y} \quad (2)$$

For the straight line velocity profile

$$\frac{\partial u}{\partial y} = \frac{u_j - u_a}{b} \quad (3)$$

and

$$\frac{\partial T_o}{\partial y} = \frac{T_{oj} - T_{oa}}{b} \quad (4)$$

Therefore

$$\tau = \rho \epsilon \frac{u_j - u_a}{b} \quad (5)$$

and

$$q = \rho c_p \epsilon \frac{T_{oj} - T_{oa}}{b} \quad (6)$$

Prandtl's expression for the apparent kinematic viscosity is

$$\epsilon = k (u_{\max} - u_{\min}) b \quad (7)$$

where k is an empirical constant.

When Eq. (7) is substituted into Eqs. (5) and (6)

$$r = \rho k (u_j - u_a)^2 \quad (8)$$

and

$$q = \rho c_p k (T_{oj} - T_{oa}) (u_j - u_a) \quad (9)$$

As it is desired to calculate shear stress and energy flux at the dividing streamline, the flow properties must be determined at that point in the mixing profile. To determine the dividing streamline velocity, a simple continuity and momentum integral analysis was applied to the two-dimensional mixing model of Fig. 4. The results are shown in Fig. 5 for equal stream temperatures; the dividing streamline velocity agrees closely with the average velocity in the mixing zone. When the effect of stream temperature difference was treated for incompressible flow, it was found that the dividing streamline velocity varies less than 20 percent from the average velocity for a range of temperature and velocity ratios up to 10. It was therefore concluded that, since the separate effects of compressibility and temperature difference were small, the assumption of the dividing streamline at the midpoint of the mixing profile was justified for the general case of compressible flow having a stream temperature difference.

At the dividing streamline the velocity, total temperature, and composition are averages of the outer stream quantities, and

$$T_{oc} = \frac{T_{oj} + T_{oa}}{2}, \quad c_{pc} = \frac{c_{pj} + c_{pa}}{2},$$

$$u_c = \frac{u_j + u_a}{2}, \quad \text{and} \quad R_c = \frac{R_j + R_a}{2}$$

(The relationship for R_c is not exact but is accurate for small differences of outer stream molecular weights.) The density on the dividing streamline, from energy and state equations, is

$$\rho_c = \frac{p}{R_c \left(T_{oc} - \frac{u_c^2}{2c_{pc}} \right)} \quad (10)$$

and the shear stress along the dividing streamline is

$$\tau_c = \rho_c k (u_j - u_a)^2 \quad (11)$$

A relationship between τ_c and q_c may be determined by solving Eqs. (5) and (6) for $\frac{\rho_c}{b}$ and equating

$$\tau_c = \frac{q_c (u_j - u_a)}{c_{pe} (T_{oj} - T_{oa})} \quad (12)$$

Thus Eqs. (11) and (12) define the shear stress and energy flux at the dividing streamline independent of the mixing zone width and only in terms of the stream conditions outside the zone.

ONE-DIMENSIONAL FLOW EQUATIONS

One-dimensional equations for momentum, continuity, and energy may now be written for each stream over a short interval, Δx (Fig. 3), and the effective shear and heat flux are assumed to be averages of those quantities calculated at the interval end points.

When the momentum equation for each stream is written in terms of the stream impulse, the sum of the momentum and pressure force for perfect gases is

$$\begin{aligned} F_{a2} &= F_{a1} + \Delta F_c - \Delta F_w - p_{avg} (A_{j2} - A_{j1}) \\ &= (A_D - A_{j2}) p_2 (1 + \gamma_a M_{a2}^2) \end{aligned} \quad (13)$$

where

$$\Delta F_w = \overbrace{\pi D_D \Delta x c_{fw}}^{\text{area}} \frac{\gamma_a}{2} p_{avg} M_{a,avg}^2 \quad (14)$$

and

$$\Delta F_c = \overbrace{\pi D_{j,avg} \Delta x}^{\text{area}} \overbrace{\left(\frac{\tau_{c1} + \tau_{c2}}{2} \right)}^{\uparrow} \quad (15)$$

and

$$\begin{aligned} F_{j2} &= F_{j1} - \Delta F_c + p_{avg} (A_{j2} - A_{j1}) \\ &= A_{j2} p_2 (1 + \gamma_j M_{j2}^2) \end{aligned} \quad (16)$$

Since τ_c and q_c act on the same surface areas, the total turbulent shear force and total energy transfer rate acting over the interval may be related using Eq. (12)

$$\Delta F_c = \frac{\Delta Q_c}{c_{p_c}} \left[\frac{u_j - u_a}{T_{oj} - T_{oa}} \right]_{avg} \quad (17)$$

The energy equation may then be written

$$\Delta Q_c = w_a c_{p_a} (T_{oa2} - T_{oa1}) = w_j c_{p_j} (T_{oj1} - T_{oj2}) \quad (18)$$

and the weight flow equation for perfect gases becomes

$$w_{a1} = w_{a2} = \frac{(A_D - A_{j2}) P_2}{\sqrt{T_{oa2}}} M_{a2} \sqrt{\frac{\gamma_a}{R_a}} \sqrt{1 + \frac{\gamma_a - 1}{2} M_{a2}^2} \quad (19)$$

and

$$w_{j1} = w_{j2} = \frac{P_2 A_{j2}}{\sqrt{T_{oj2}}} M_{j2} \sqrt{\frac{\gamma_j}{R_j}} \sqrt{1 + \frac{\gamma_j - 1}{2} M_{j2}^2} \quad (20)$$

By substitution, the preceding set of equations may be reduced to six unknowns, P_2 , A_{j2} , M_{a2} , M_{j2} , T_{oa2} , and T_{oj2} , with the transfer quantities ΔF_c , ΔF_w , and ΔQ_c expressed in terms of Δx and averages over the interval of the six basic variables. The set of equations could be written for solution with a digital computer; however, an iteration procedure amenable to hand calculation has been developed.

The iteration procedure can be greatly facilitated by defining the following parameters. Thus Eqs. (13) and (19) are combined

$$\frac{F_{a2}}{w_a \sqrt{T_{oa2}}} = \frac{1 + \gamma_a M_{a2}^2}{M_{a2} \sqrt{\frac{\gamma_a}{R_a}} \sqrt{1 + \frac{\gamma_a - 1}{2} M_{a2}^2}} \quad (21)$$

and similarly Eqs. (16) and (19) are combined

$$\frac{F_{j2}}{w_j \sqrt{T_{oj2}}} = \frac{1 + \gamma_j M_{j2}^2}{M_{j2} \sqrt{\frac{\gamma_j}{R_j}} \sqrt{1 + \frac{\gamma_j - 1}{2} M_{j2}^2}} \quad (22)$$

Equations (21) and (22) now present expressions for the parameter, $F/w\sqrt{T_o}$ which is proportional to F/F^* of the one-dimensional Fanno analysis for duct flow and Mach number. There are two solutions, subsonic and supersonic, for each value of $F/w\sqrt{T_o}$, but since the

method is concerned with a smooth transition of axial properties, the Mach numbers at the end of the calculation interval will be supersonic for initial supersonic conditions and subsonic for initial subsonic conditions. Curves of the supersonic $F/w\sqrt{T_0}$ vs Mach number relationship for air and for typical rocket exhaust products are shown in Fig. 6. Details of the iteration procedure are given in the Appendix.

EMPIRICAL MIXING PARAMETERS

In the development of the simplified mixing theory, only one empirical constant, k , was defined, other than the wall skin friction coefficient which is well known. An extensive literature survey revealed considerable discrepancy in the definition of mixing parameters by various investigators. Also, these empirical parameters are usually evaluated by correlating a particular theoretical approach with experimental data, and thus they reflect the peculiarities of the particular analysis. In addition, most of the available data is for the case of constant pressure mixing; however, it was felt that turbulent parameters would not differ greatly for the nonconstant pressure case and that information for the constant pressure case would help predict magnitudes and trends of the empirical coefficient for the problem of interest. In any case, it was necessary to determine proper values of k for the simplified theory by correlation with experiment.

Constant Pressure Plane Mixing with Zero Secondary Velocity

For constant pressure mixing between a uniform stream and the surrounding ambient gas, various turbulent exchange parameters have been used. Szablewski (Ref. 2) uses a similarity parameter, σ , which by definition, is inversely proportional to the mixing zone spreading rate, db/dx . The various turbulent exchange parameters are also related by (Ref. 2)

$$\sigma = \frac{1}{\sqrt{2k} \, db/dx} \quad (23)$$

Since σ is inversely proportional to db/dx , then k is directly proportional to db/dx .

The incompressible value for spreading rate has been established by Tollmien (Ref. 3) as $db/dx = 0.255$ for a uniform stream mixing with a surrounding ambient gas. The value of $\sigma = 12$ also has been established for this case. Then Eq. (23) yields $k = 0.0136$. (Szablewski (Ref. 2) computed the value $k = 0.0158$.)

It is generally known that increasing stream Mach number decreases the rate of spreading. Data obtained at Princeton (Refs. 4 and 5) and at NACA (Ref. 6) are plotted in Fig. 7 for zero secondary flow. There appears to be an almost linear decrease of db/dx , and therefore k , with increasing Mach number.

Finite Secondary Velocity

For finite secondary stream velocity, u_a , Eq. (23) becomes (Ref. 2)

$$\sigma = \frac{1}{\sqrt{2k} (db/dx)_{u_a=0} (1 - u_a/u_j)} \quad (24)$$

and db/dx varies as $(1 - u_a/u_j)$; k is not affected by finite secondary stream velocities. Spreading rate data from Princeton (Ref. 4) agree well with the predicted trend of db/dx with u_a/u_j .

Effect of Stream Temperature Difference

Very little experimental data have been published concerning the effect of stream temperature differences on turbulent mixing rates. Data from Princeton (Ref. 4) for moderate temperature difference and subsonic flow indicate that increasing the primary stream total temperature tends to increase the spreading rate; however, insufficient data are available to fully define the trend.

Constant Pressure Mixing of an Axisymmetric Jet

For fully developed mixing of an axisymmetric free jet where there is a decay of centerline velocity, somewhat different values of k have been observed. Szablewski (Ref. 7) determined a value of $k = 0.0105$ to correlate his theory with experimental data for the diffusion of a fully developed jet. Warren (Ref. 8) correlated experimental data with his integral equation theory for the spreading of a fully developed axisymmetric jet with zero secondary velocity. Warren's incompressible value for k is 0.0217; the difference between this and the value 0.0105 reported by Szablewski reflects differences in the methods of analysis. Warren's results for k are shown in Fig. 8 plotted against initial primary jet Mach number along with the empirical relationship used to correlate the data.

EXPERIMENTAL INVESTIGATION

EXPERIMENTAL APPARATUS

Open-Circuit Tunnel

The experimental program was conducted in a small open-circuit wind tunnel installed in the Propulsion Research Laboratory (T5C-2) of the RTF. A schematic of the tunnel is shown in Fig. 1. Tunnel air was supplied from the RTF air machinery at total pressures up to 40 psia. A direct-fired heater using JP-4 was installed at the inlet to the plenum chamber to heat the tunnel air up to a maximum of 1200°F. The flow was accelerated in an 8.1-in. -diam, Mach number 3.07, axisymmetric nozzle. A 2.83-in. -diam centerbody designed by the method of Ref. 9 to provide uniform annular flow past the model base was installed in the tunnel nozzle. The diffuser duct used for this investigation was 8.1 in. in diameter and had a length of 10 diameters. The construction of the downstream flange caused a small (approximately 4 percent) area restriction at the end of the duct.

A small water-cooled rocket motor utilizing inhibited red fuming nitric acid (IRFNA) and unsymmetrical dimethyl hydrazine (UDMH) as propellants was mounted at the downstream end of the centerbody. The flow configuration past the end of the centerbody essentially simulated the flow past a single rocket missile base in free flight. A summary of principal tunnel and rocket dimensions and operating parameters is presented in the Table.

Rocket Instrumentation

Rocket chamber pressure and injector head pressures were measured by strain-gage transducers and recorded on a recording oscillograph. Fuel and oxidizer flow rates were measured by turbine-type flowmeters and recorded on the oscillograph. The sequence of propellant valve operation was also recorded on the oscillograph.

Rocket chamber pressure and propellant system tank pressures were measured by strain-gage transducers and recorded on strip charts for inspection during rocket operation. Fuel, oxidizer, and cooling water flow rates were also monitored visually by digital time and frequency meters.

Aerodynamic Instrumentation

Pressures necessary for setting tunnel operating conditions (tunnel total pressure, RTF exhaust pressure, spray section pressure, and

nozzle exit static pressure) were measured on high accuracy diaphragm gages. These pressures, as well as the static pressure distribution from the nozzle exit to the spray section, were also measured by mercury manometers referenced to atmosphere and recorded photographically. The estimated error of the pressures measured on the manometers is ± 0.05 psi.

Mach number distribution over the nozzle annulus was determined during initial calibration runs by a 5-station total pressure rake installed just upstream of the end of the centerbody. The pressures were measured with mercury manometers and recorded photographically.

Tunnel total temperature was measured by iron-constantan thermocouples read out visually on temperature-calibrated galvanometers.

EXPERIMENTAL PROCEDURE

The following procedure was used to obtain the data on supersonic two-stream mixing:

1. Tunnel flow with a very low back pressure was established so that supersonic flow existed in the entire straight section of the diffuser.
2. The rocket fuel and oxidizer valves were opened, and ignition was accomplished when the hypergolic propellants were injected into the chamber.
3. The rocket firing continued for 20 to 30 sec to allow stabilization of the instrumentation. Pictures of the manometers were taken automatically every three seconds during the rocket firing to obtain a record of the manometer response.

DISCUSSION OF RESULTS

To verify that the simplified theory gives an adequate explanation of the bounded two-stream mixing process, correlations were made with Mikhail's experimental data (Ref. 1) for incompressible flow. (A schematic of Mikhail's experimental apparatus is shown in Fig. 9.) The simplified theory was also correlated with Mikhail's more elaborate integral equation theory for incompressible mixing. The one-dimensional flow equations were written in a form applicable to incompressible flow, and calculations were carried out using various values of the turbulent exchange parameter, k , for the initial conditions shown in Fig. 9. It was

found that the best correlation was obtained using $k = 0.010$, which is essentially the value used by Szablewski (Ref. 7) for the constant pressure diffusion of a fully developed incompressible jet.

A comparison of Mikhail's experimental axial pressure distribution with theoretical distributions calculated from the simplified theory and from Mikhail's theory is shown in Fig. 10. Both theoretical pressure distribution curves differ considerable from the experimental curve for the first eight duct diameters. Mikhail explains this discrepancy as caused by a low pressure region in the center of the primary jet which extends about the first eight duct diameters; however, this explanation violates the assumption in both theories of constant pressure across any section. Other investigators have also found the same shape for the experimental pressure distribution curve, and it appears typical for low velocity jet pump configurations.

A comparison of the experimental centerline velocity distribution with theoretical distributions calculated using the two theories is shown in Fig. 11. The simplified theory correlates the experimental data poorly for the first six to eight duct diameters but shows acceptable agreement farther downstream. Mikhail's theory gives excellent correlation with experimental velocity distribution data since each of his three empirical mixing parameters was determined by matching these theoretical and experimental centerline velocity distributions. Although the simplified theory presented in this report only approximately predicts experimental velocity and pressure distributions for the first six to eight duct diameters, the behavior farther downstream is predicted closely and agrees well with the predictions of Mikhail's more elaborate theory. It can be concluded that the simplified theory gives a reasonably accurate overall picture of the mixing process and may be used to optimize low velocity mixing tube configurations.

COMPRESSIBLE TWO-STREAM FLOW

A basic assumption of the simplified analysis is that the two streams are initially uniform and parallel. However, the actual wind tunnel configuration used was not designed specifically to study the mixing problem and violates this assumption to a certain extent. A schematic of the model base geometry and flow just downstream of the rocket is shown in Fig. 12. To apply the simplified mixing theory to the initial flow conditions of the actual tunnel, it was assumed that at some section downstream the two streams become uniform and parallel. The flow conditions at this hypothetical section were calculated assuming that each

stream undergoes an isentropic area change to equalize pressures in the two streams. These flow conditions are then used as the initial conditions for the mixing analysis.

Axial duct static pressure distributions for the Mach number 3.07 tunnel operating at a tunnel total temperature of 1100°R are shown in Figs. 13a, b, and c. The experimental rocket-off distributions shown indicate that the tunnel back pressure was sufficiently low to allow supersonic flow over the entire duct. In the case of rocket-on operation, the high initial duct static pressure just downstream of the model was caused by the impingement and reflection at the wall of the shock formed at the intersection of the two streams. After about two to three duct diameters, the magnitude of the disturbances was considerably decreased, and a reasonable average static pressure distribution may be drawn through the experimental points. The initial shock increased in strength with lower tunnel static pressure (increased underexpansion of the rocket nozzle), and the assumption of uniform initial stream conditions was violated to a greater extent as tunnel pressure decreased.

When the simplified mixing theory was applied to the experimental data (Fig. 13), it was found that the best correlation was obtained for $k = 0.010$, the value used for incompressible flow. Apparently the trend of decreasing k with increasing Mach number is equally opposed by the trend of increasing k with increasing stream temperature difference for the particular experimental parameters.

Experiments were also made with a lower tunnel total temperature (500-600°R). In this case thermal choking occurred in the 10-diam duct, even though the back pressure was maintained at a value lower than the duct exit pressure. The choking caused the tunnel nozzle flow to break down, and the resulting static pressure distributions were indistinguishable from those which occurred when the mixing of the streams was induced by a fully developed duct shock system. The simplified theory was applied to the cases where choking was observed, and the resulting outer stream axial Mach number distributions are shown in Fig. 14. At the 10-diam station, theoretical values for M_a are 1.25 and 1.48, respectively, for tunnel pressures of 19.9 and 37.8 psia. The 4-percent area reduction at the downstream flange, however, would cause choking to occur at approach Mach numbers of 1.2 or 1.3 rather than 1.0. Also, the theoretical Mach number, M_a , is based on the discontinuous velocity profile in Fig. 3 and represents an average Mach number for the outer airstream. The total pressure loss caused by the shocks formed at the intersection of the initially non-uniform streams would also cause choking to occur at theoretical values of M_a somewhat greater than 1.0. Theoretical Mach number distributions for 1100°R tunnel total temperature where

no choking occurred are shown in Fig. 15; in this case the theoretical value of M_a is approximately 2.0 at the 10-diam station. It can thus be concluded that the simplified theory indicates the correct trends of outer stream Mach number and axial static pressure and that the essential features of the mixing process are preserved in the analysis.

CONCLUSIONS

The following conclusions may be drawn from the portion of the two-stream diffuser investigation reported herein:

1. The simplified treatment of constant area two-stream mixing preserves the essential features of the process and correlates well with other approaches for the case of low velocity flow.
2. The simplified theory explains the general behavior, with respect to Mach number and static pressure, of the constant area mixing of two supersonic streams.
3. Thermal choking in a constant area duct can be caused by turbulent mixing between a supersonic rocket stream and a surrounding supersonic airstream, even when the duct back pressure is maintained at a low value.

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APPENDIX

ITERATION PROCEDURE FOR MIXING ANALYSIS

For the general case of bounded mixing of a rocket stream with a surrounding airstream, the following iteration procedure was used:

A. First Iteration

All parameters at the beginning of the calculation interval are given. Choose ΔQ_c which defines T_{oa2} and T_{oj2} from Eq. (18). As a first approximation assume that the average transfer quantities over the interval are those calculated at the initial section.

1. Calculate r_{c1} from Eq. (11) using proper value of the empirical parameter k .
2. Use $\left[\frac{u_{j1} - u_{a1}}{T_{oj1} - T_{oa1}} \right]$ for $\left[\frac{u_j - u_a}{T_{oj} - T_{oa}} \right]_{avg}$ and calculate ΔF_c from Eq. (17).
3. Use r_{c1} for $r_{c_{avg}}$, D_{j1} for $D_{j_{avg}}$, and solve for Δx from Eq. (15).
4. Calculate ΔF_w from Eq. (14) using initial stream conditions and Δx calculated in step 3.
5. Neglect the $p\Delta A$ force and calculate F_{a2} and F_{j2} from Eqs. (13) and (16).
6. Evaluate the parameters $F_{a2} / w_a \sqrt{T_{oa2}}$ and $F_{j2} / w_j \sqrt{T_{oj2}}$ and determine M_{a2} and M_{j2} from a plot similar to Fig. 6 for the appropriate gas properties.
7. Calculate u_{a2} and u_{j2} , and determine $\left[\frac{u_j - u_a}{T_{oj} - T_{oa}} \right]_{avg}$ for the interval.
8. Solve Eqs. (13) and (16) simultaneously for p_2 and A_{j2} .
9. Calculate ρ_{c2} and evaluate r_{c2} from Eq. (11)

B. Second Iteration

1. Recalculate ΔF_c from Eq. (17) using average quantities found in the first iteration.
2. Solve for Δx from Eq. (15), again using average quantities over the interval.
3. Calculate ΔF_w from Eq. (14) using average outer stream conditions and Δx calculated in step 2.
4. Calculate $p_{avg} \Delta A$ force from the first iteration and evaluate F_{a2} and F_{j2} .

5. Repeat steps (6) to (8) of first iteration.
6. Solve for Δx from Eq. (15).

The iteration procedure is repeated until the values for $F_{a,2}$ and $F_{j,2}$ calculated at the end of an iteration agree, to the desired accuracy, with those used at the beginning of that iteration.

After the flow properties at the end of the initial interval are calculated to the desired accuracy, the process is repeated, using the end conditions of the first interval as initial conditions for the next. It was found, for the parameters of the experimental apparatus, that increments of $T_{0,2}$ of 100°R gave adequate accuracy.

If any simplifying condition exists, such as no wall friction, constant stream temperature, or incompressible flow, the iteration procedure is greatly simplified. It is also possible to include duct area change; however, the iteration procedure is more complicated.

TABLE

TUNNEL DIMENSIONS AND OPERATING PARAMETERS

| | |
|---|-----------------------------|
| Tunnel Nozzle Exit Diameter | 8.1 in. |
| Centerbody Diameter (nozzle exit plane) | 2.83 in. |
| Rocket Nozzle Exit Diameter | 1.25 in. |
| Rocket Nozzle Area Ratio | 8 |
| Rocket Nozzle Half Angle (conical nozzle) | 15 deg |
| Nominal Rocket Chamber Pressure | 520 psia |
| Nominal Rocket Weight Flow | 0.52 lb _m /sec |
| Nominal Rocket Oxidizer-Fuel Ratio | 2.8 |
| Nominal Rocket Vacuum Thrust (calculated) | 149 lbf |
| Theoretical Rocket Chamber Temperature | 5500°R |
| Theoretical Rocket Exhaust Gas Specific Heat (c_{p_j}) | 0.46 Btu/lb _m °R |
| Theoretical Rocket Exhaust Gas Molecular Weight | 23.3 |
| Theoretical Rocket Exhaust Gas Ratio of Specific Heats (γ) | 1.23 |

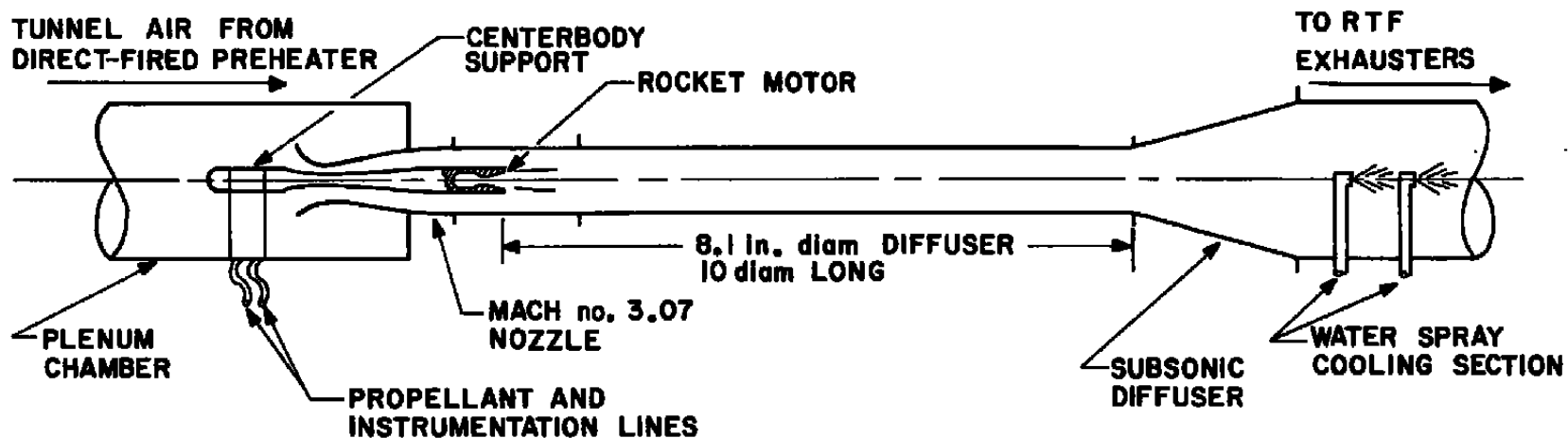


Fig. 1 Schematic of Tunnel for Two-Stream Diffuser Investigation

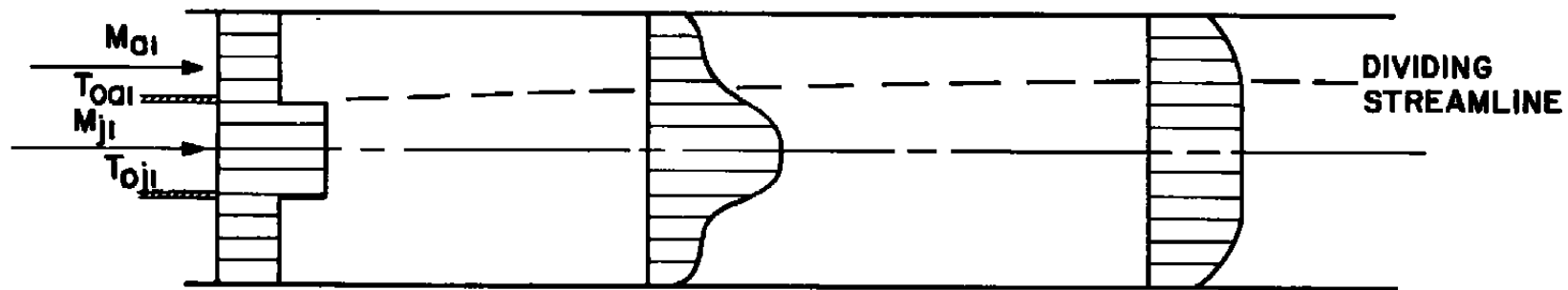


Fig. 2 Schematic of Velocity Profiles for Coaxial Stream Duct Flow

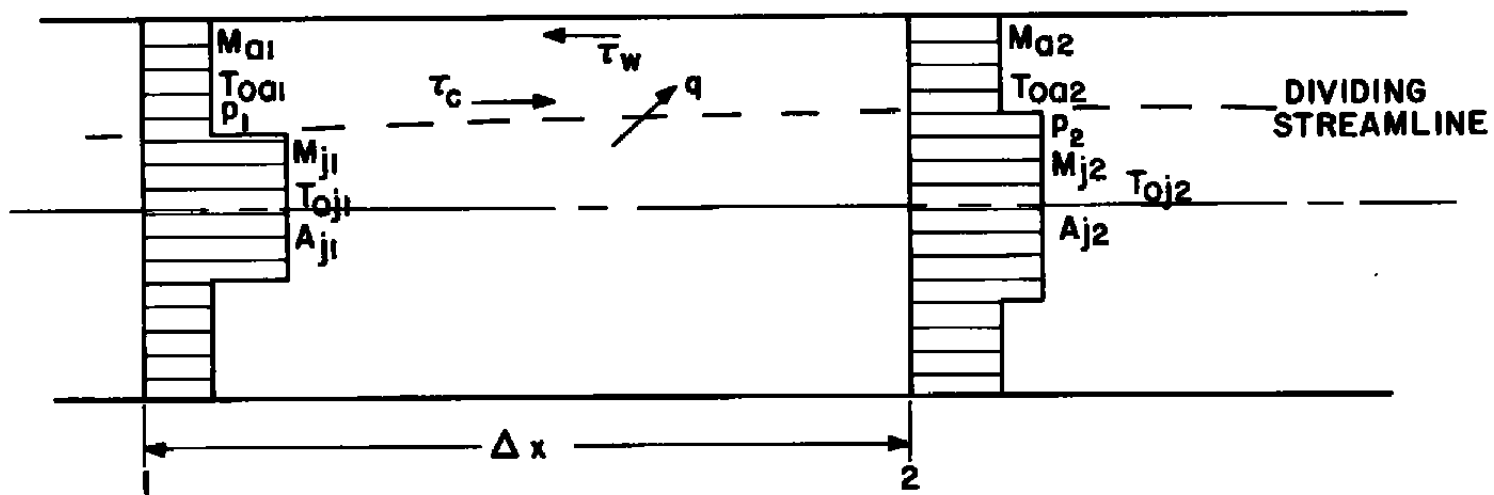


Fig. 3 Approximation of Actual Velocity Profiles for Coaxial Duct Flow

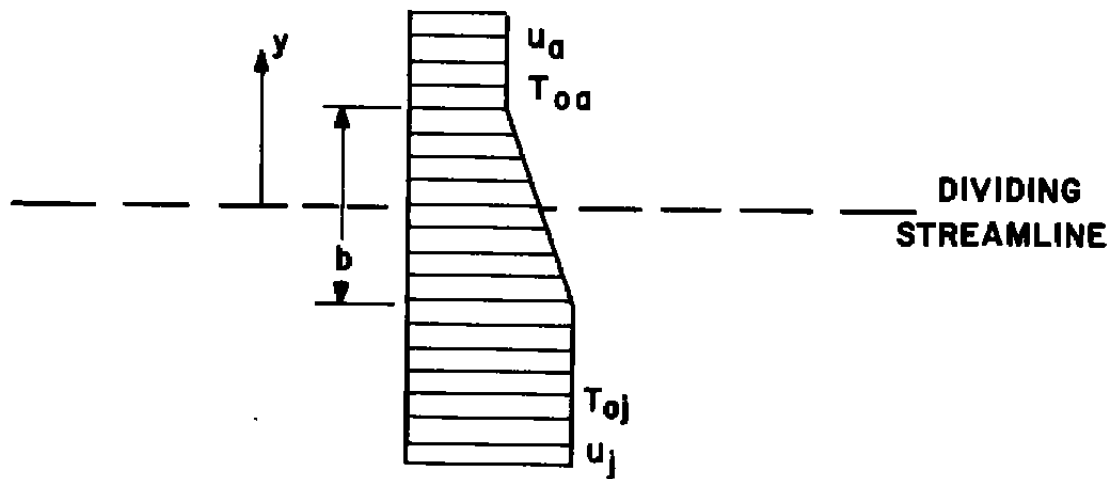


Fig. 4 Mixing Model for Calculation of Local Rates of Heat and Momentum Transfer

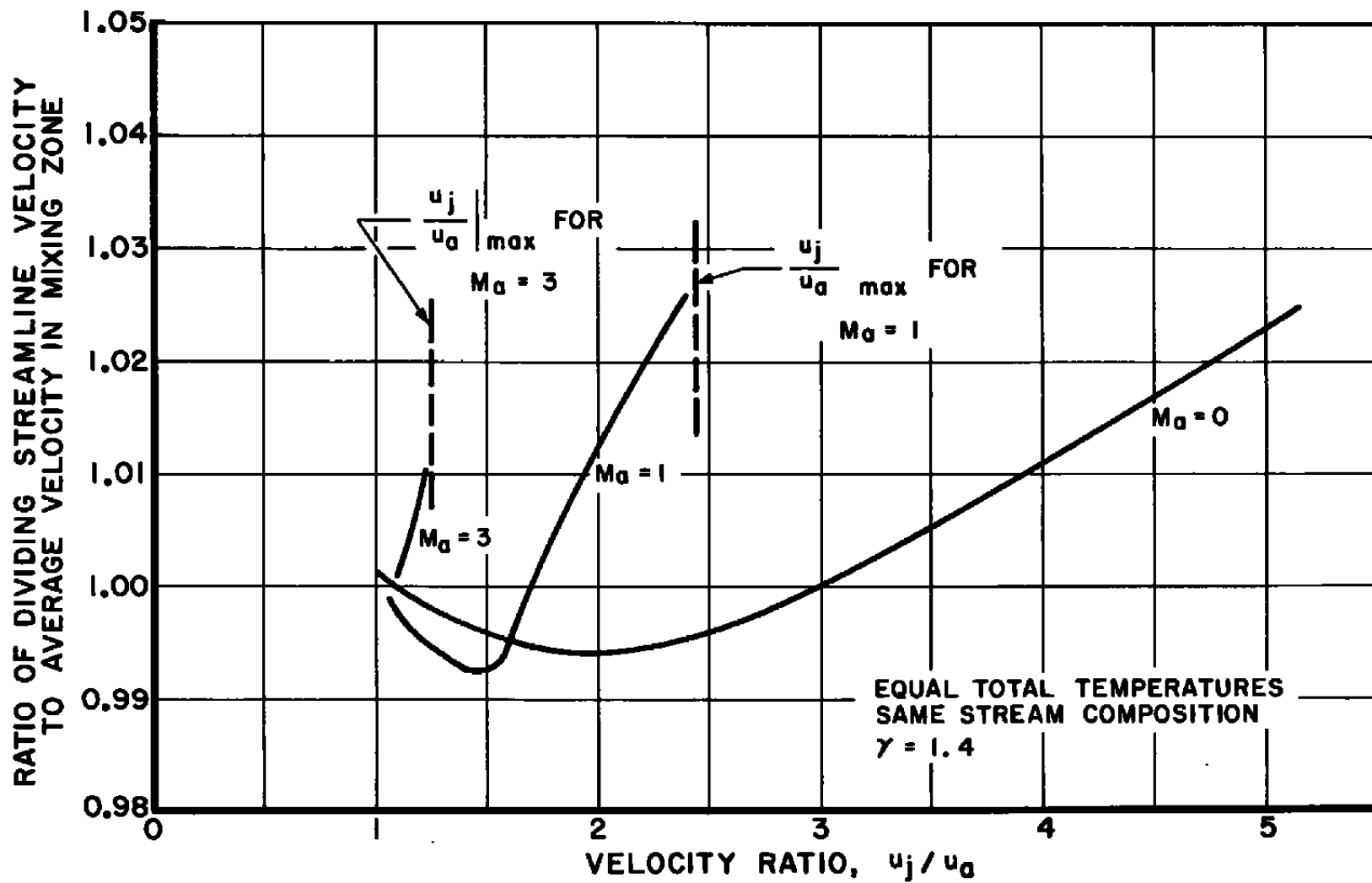


Fig. 5 Dividing Streamline Velocity Ratio

F10R

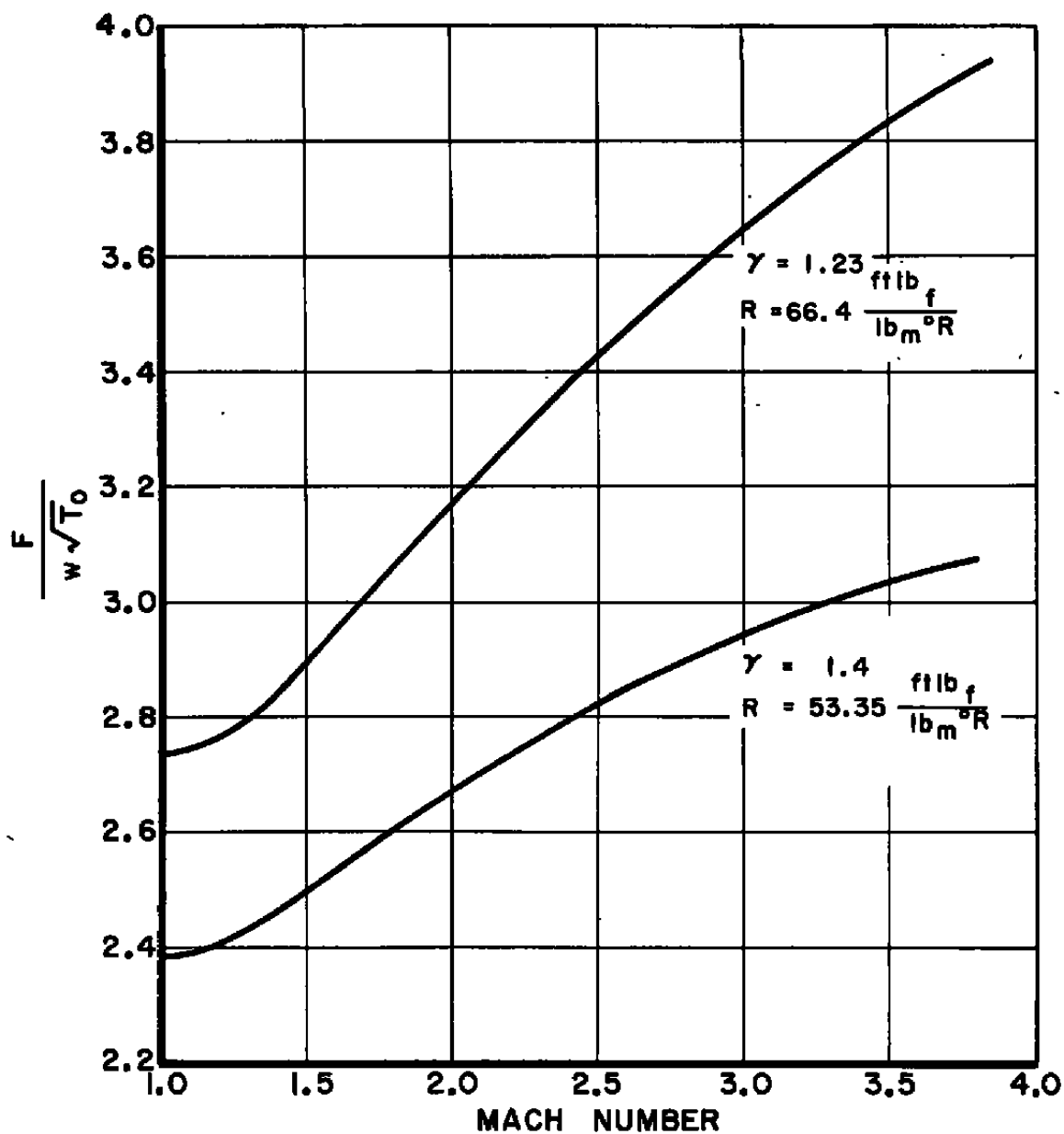


Fig. 6 Mach Number Function for Air and Typical Rocket Products

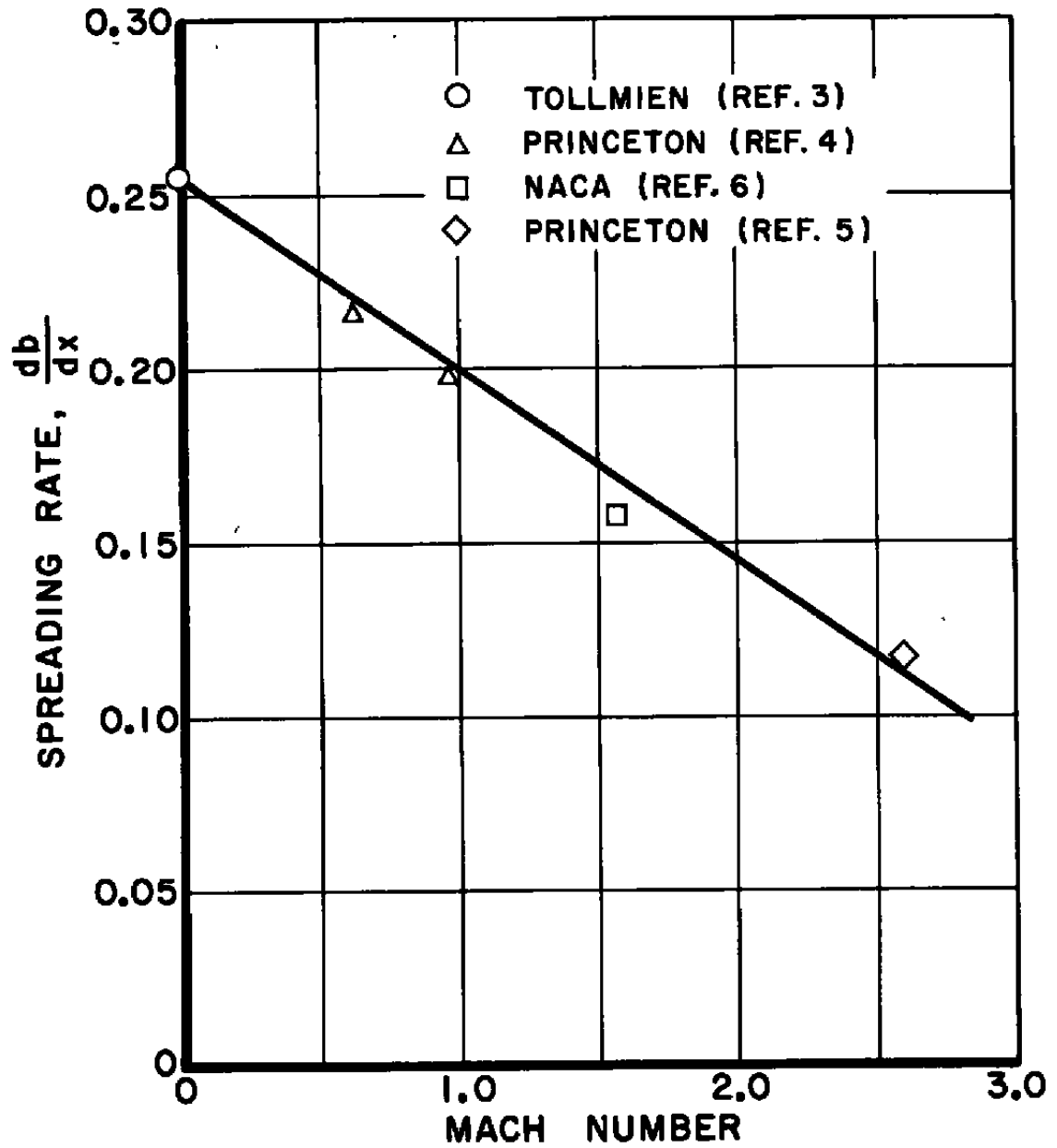


Fig. 7 Variation of Mixing Zone Spreading Rate with Mach Number for Zero Secondary Velocity

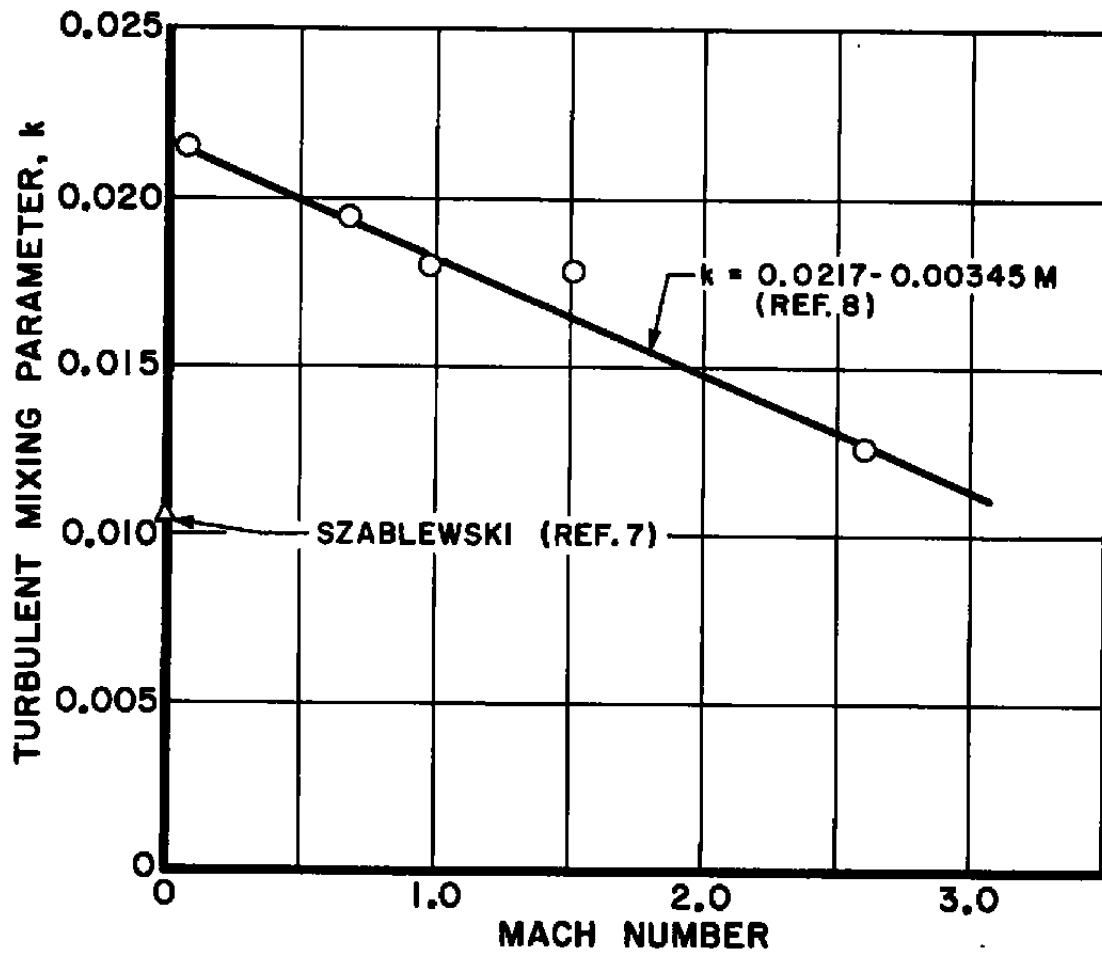


Fig. 8 Variation of Turbulent Mixing Parameter with Mach Number for Fully Developed Mixing of an Axisymmetric Jet

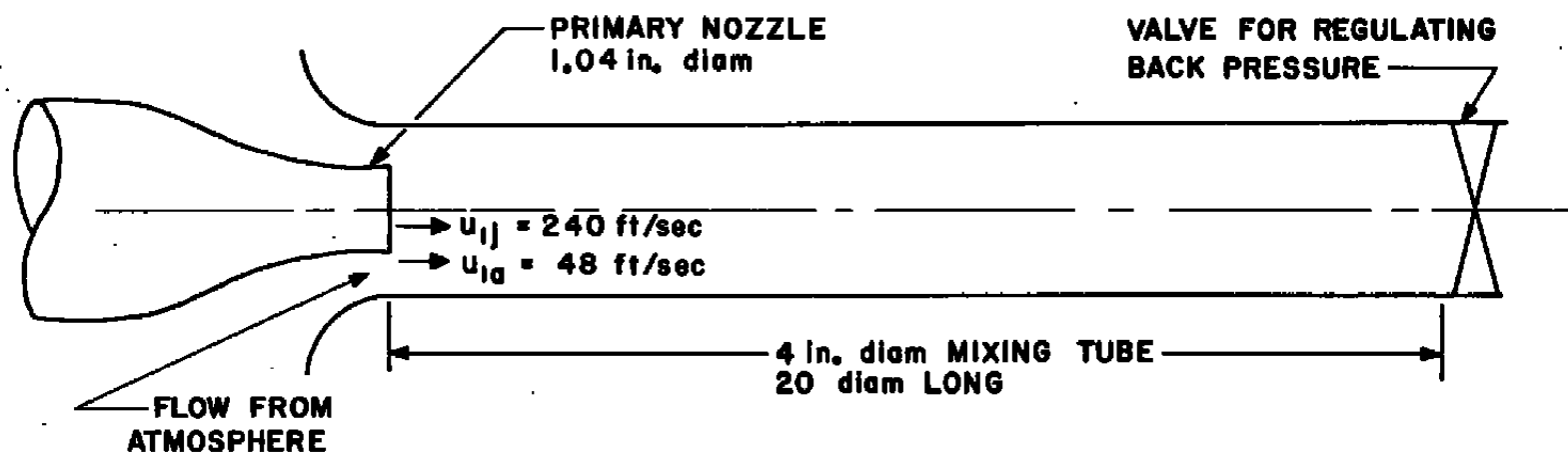


Fig. 9 Schematic of Mikhail's Experimental Apparatus

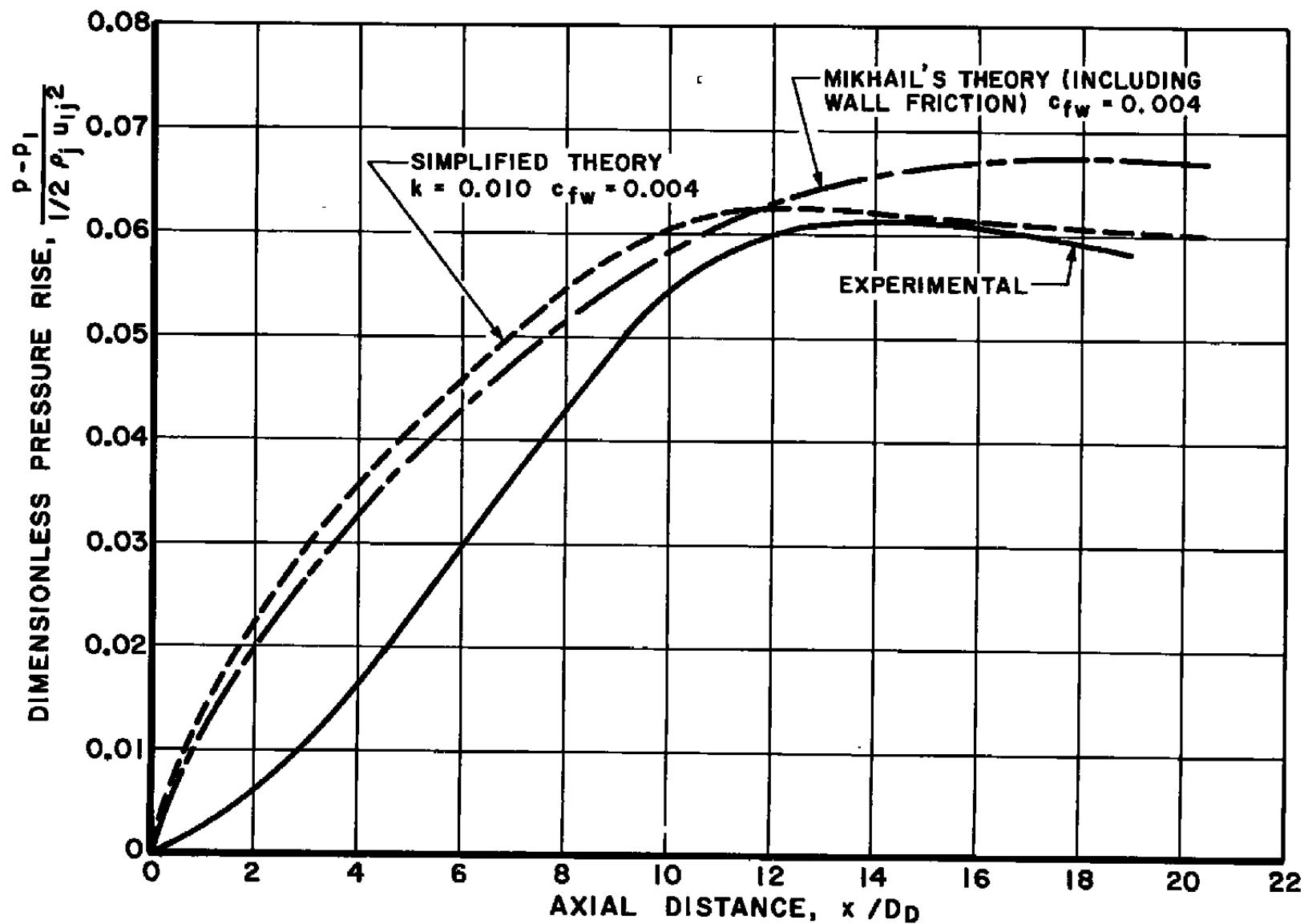


Fig. 10 Comparison of Theory with Mikhail's Experimental Data

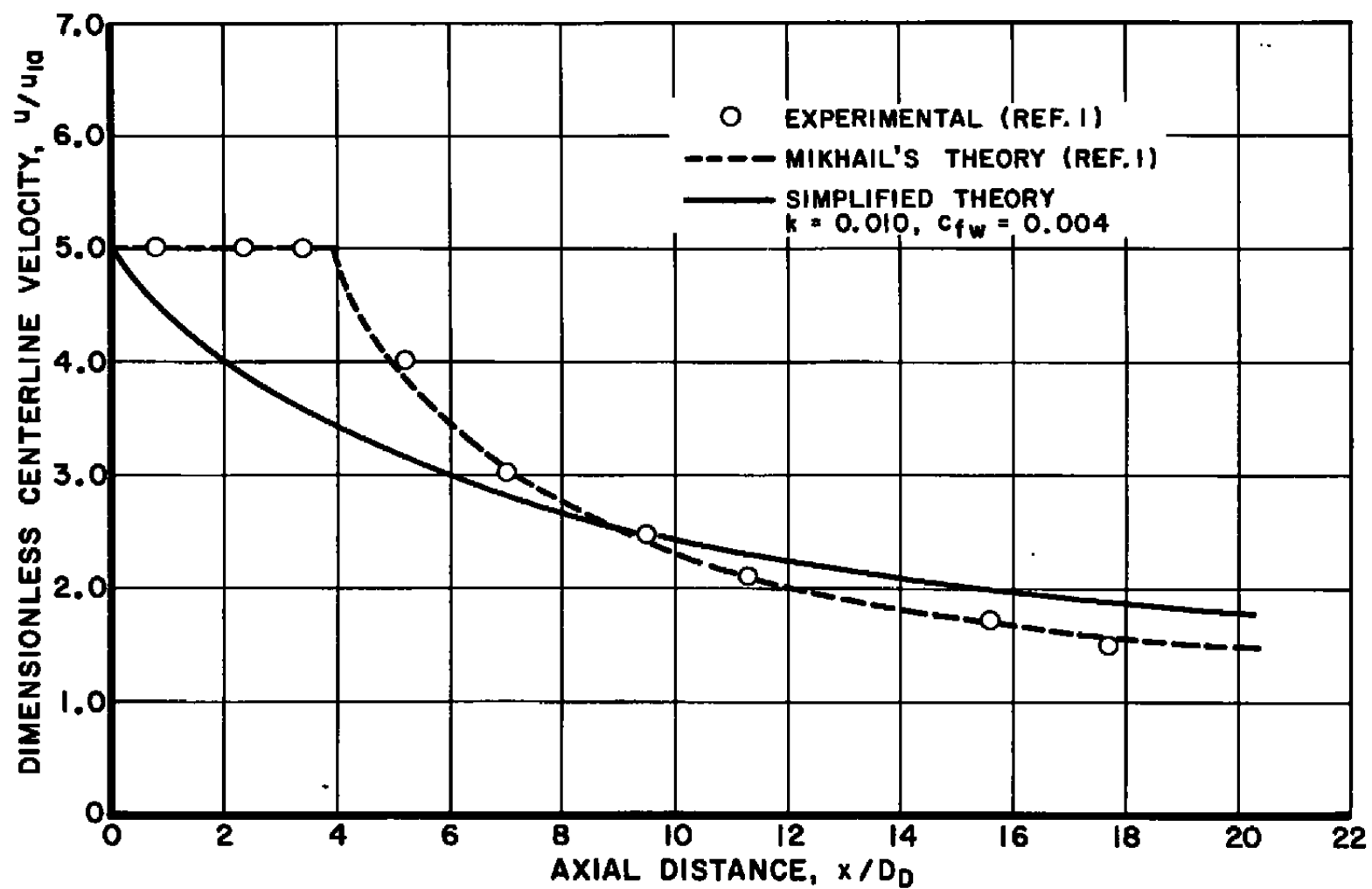


Fig. 11 Comparison of Theoretical and Experimental Centerline Velocity Distribution

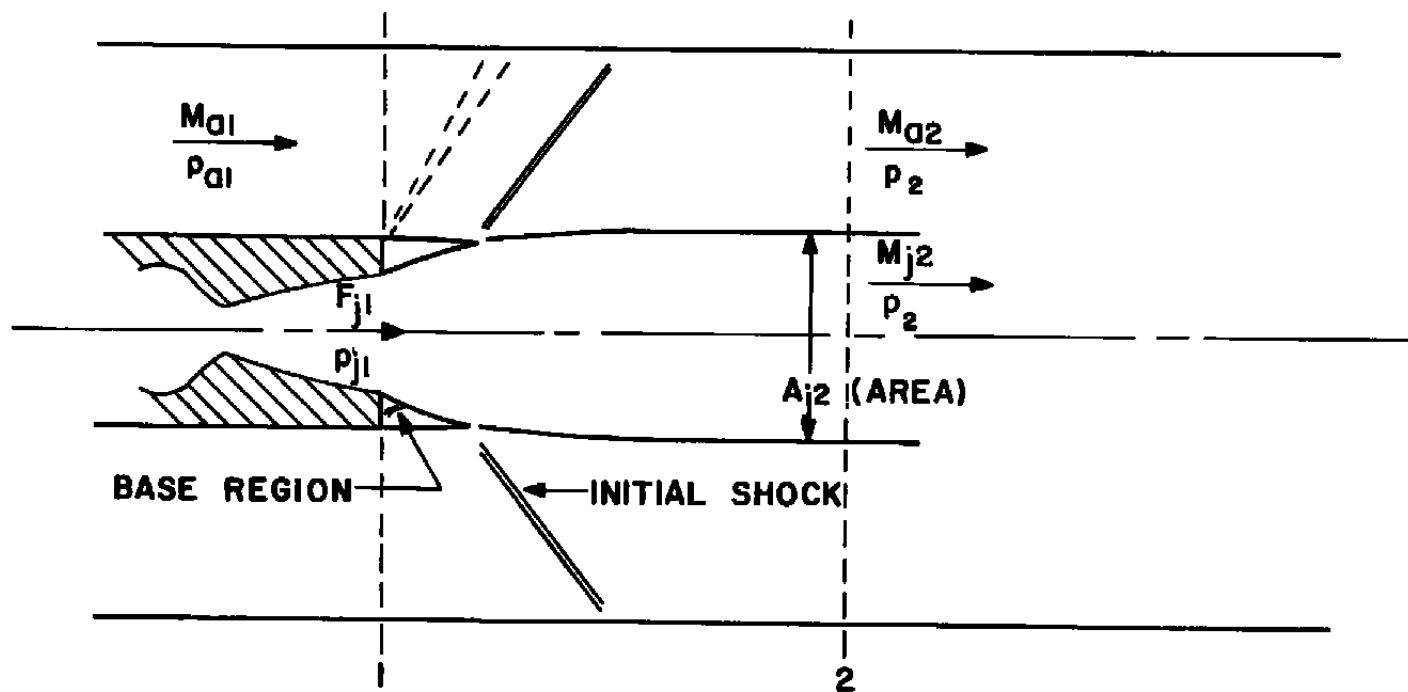
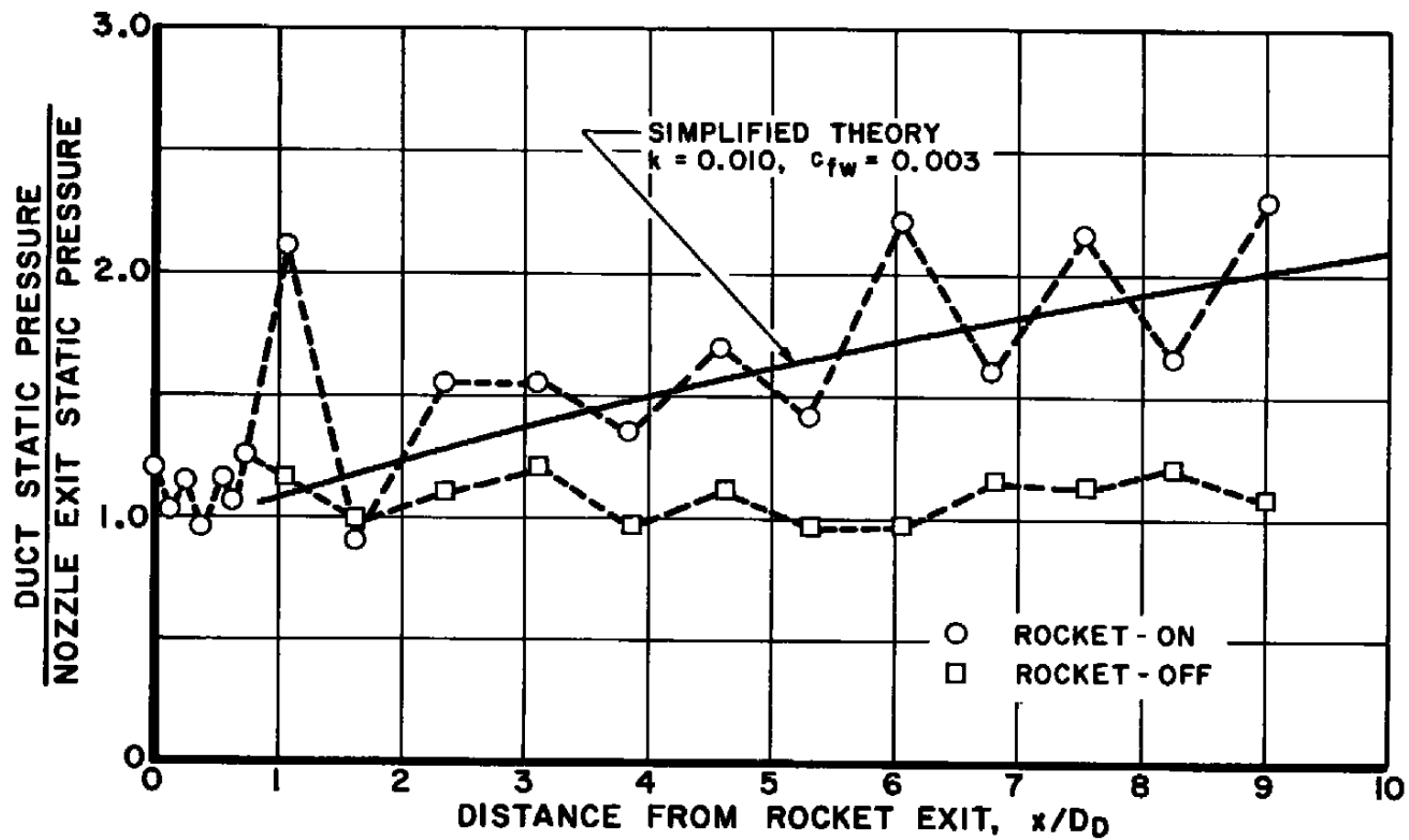
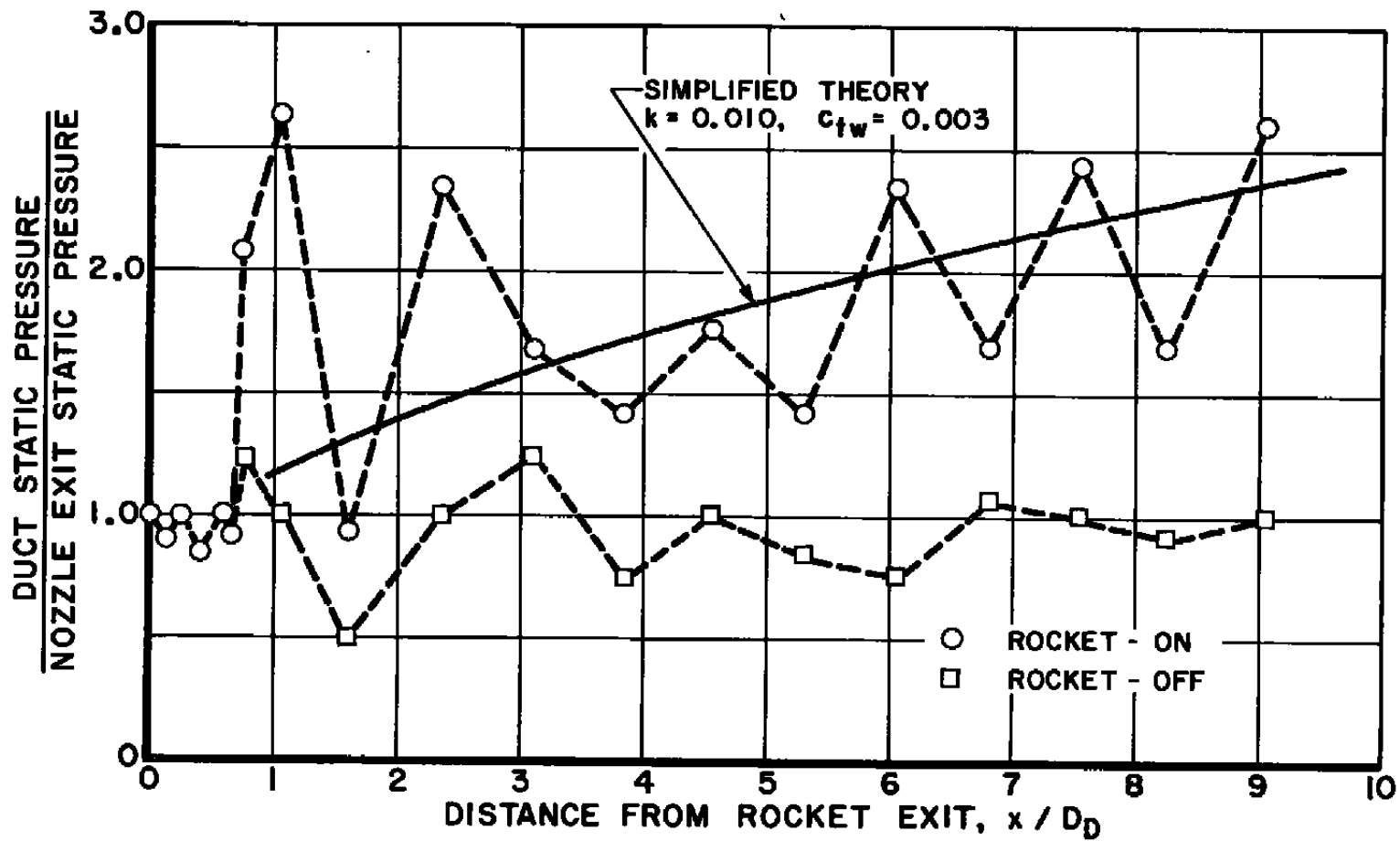


Fig. 12 Schematic of Supersonic Jet Expanding Into Supersonic Stream



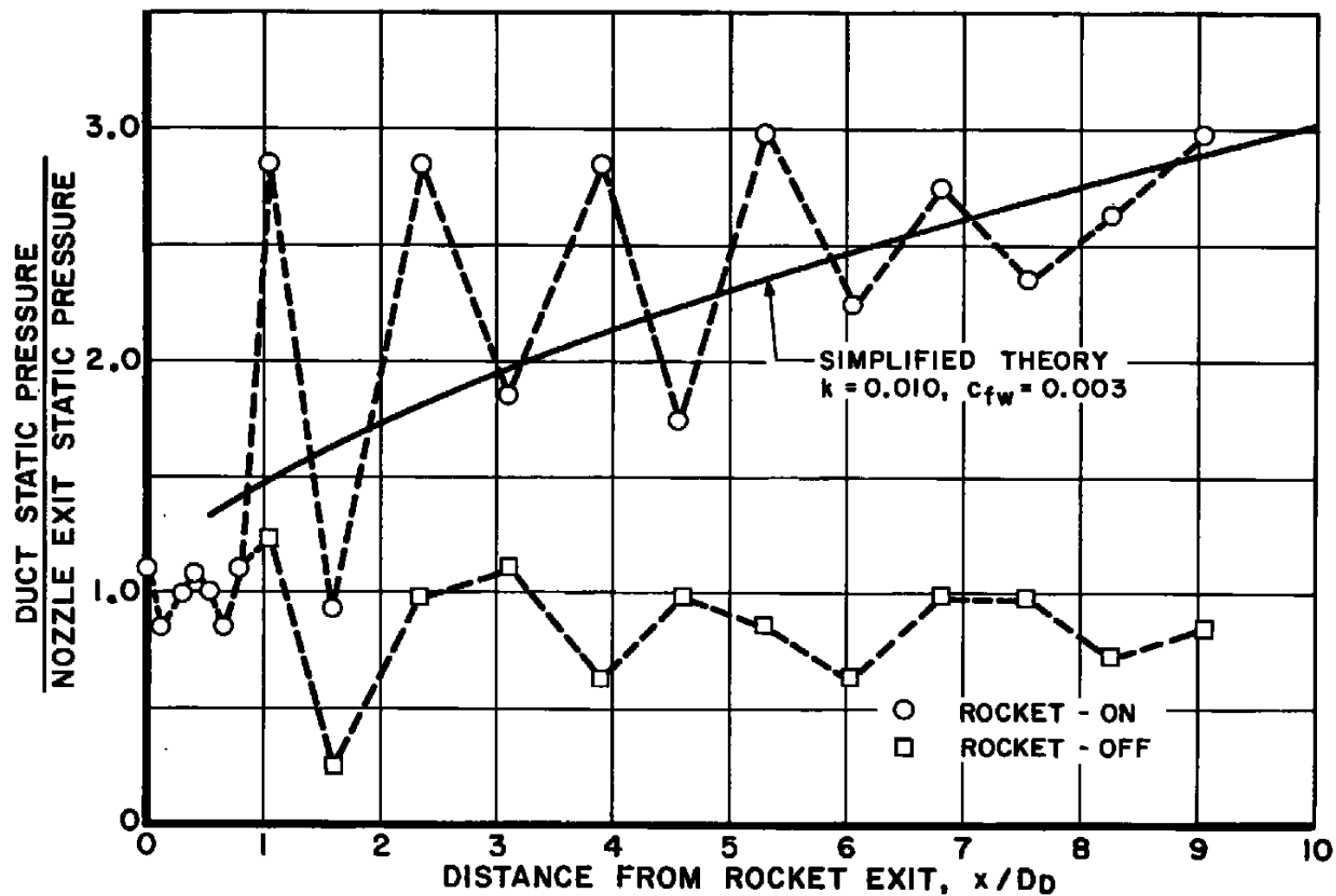
a. $p_{00} = 40.8$ psia

Fig. 13 Axial Static Pressure Distribution for Mach Number 3.07 Tunnel at $T_{00} = 1100^\circ\text{R}$



b. $p_{00} = 23.5$ psia

Fig. 7 Continued



c. $P_{00} \approx 16.0$ psia

Fig. 7 Concluded

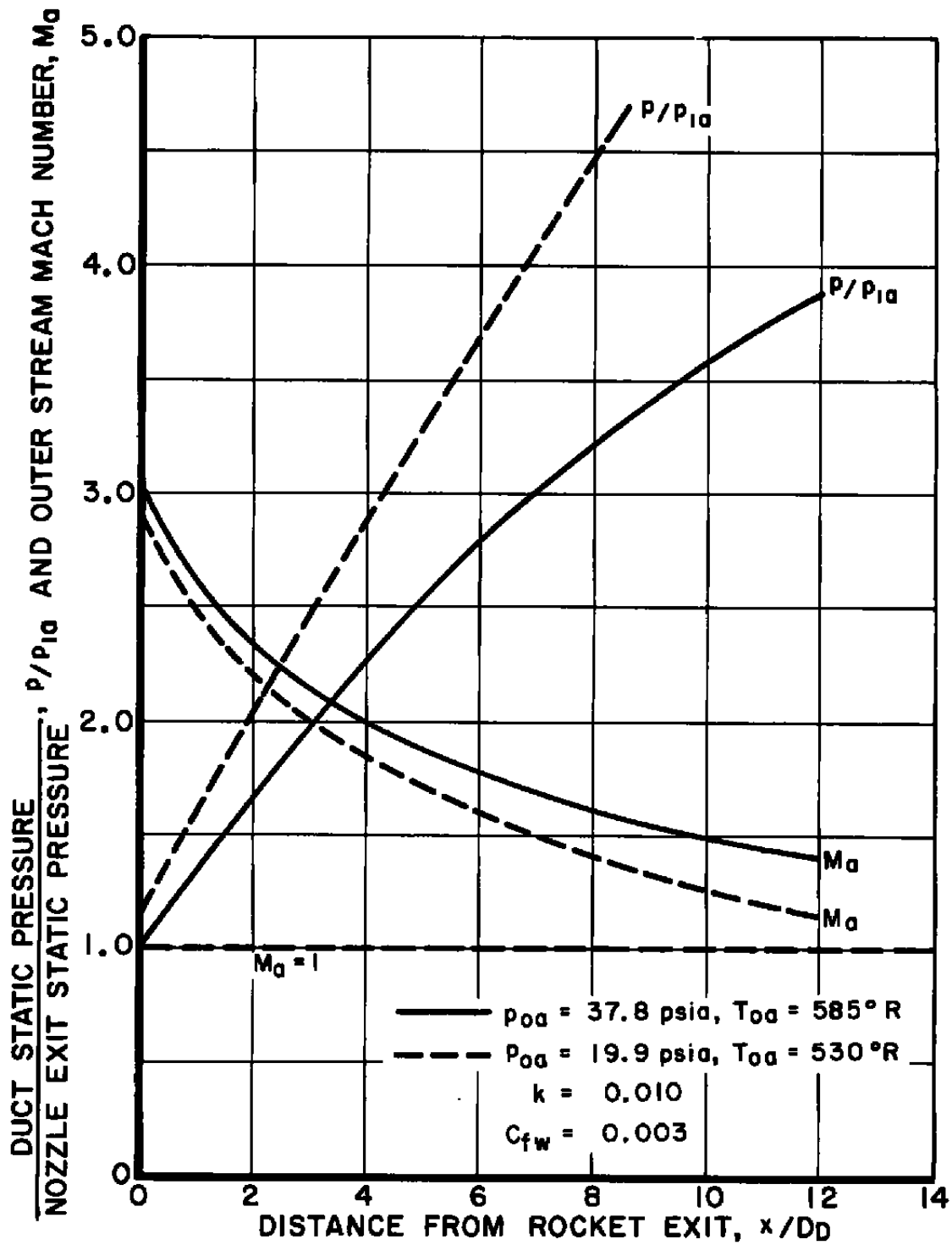


Fig. 14 Theoretical Pressure and Mach Number Distributions for Mach Number 3.07 Tunnel

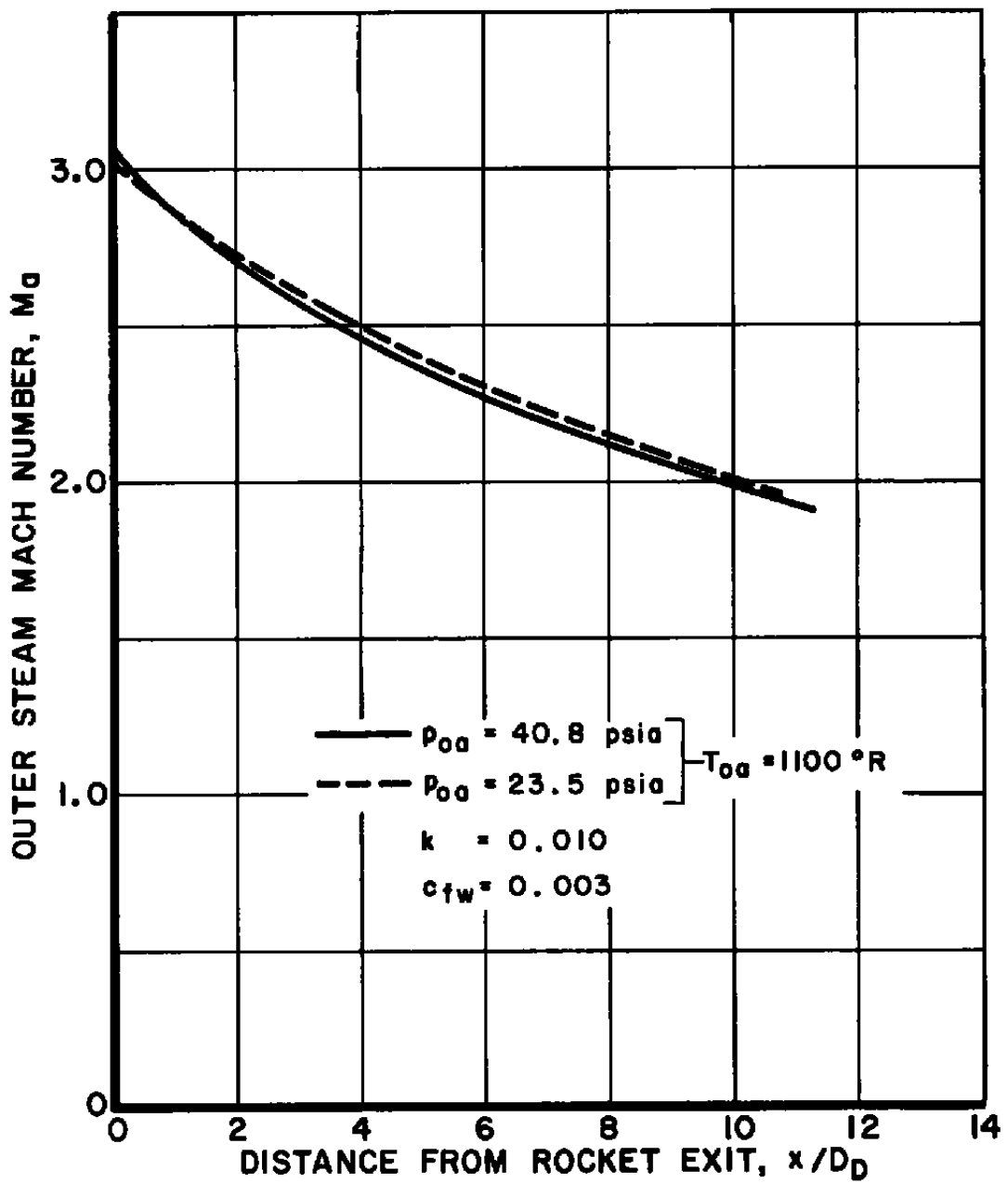


















Fig. 15 Theoretical Outer Stream Mach Number Distribution for Mach Number 3.07 Tunnel

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| <p>Arnold Engineering Development Center Arnold Air Force Station, Tennessee</p> <p>Rpt. No. AFDC-TR-61-18. CONSTANT AREA MIXING OF NON-ISOENERGETIC COAXIAL COMPRESSIBLE STREAMS. January 1962, 43 p. incl 9 refs., illus., table. Unclassified Report</p> <p>As part of a general investigation of two-stream supersonic diffusers, a theory was developed for the bounded turbulent mixing between coaxial high velocity streams of different composition and temperature. The usual assumption of negligible axial pressure gradient is invalid in this case because the outer stream is bounded by a cylindrical duct. A highly simplified flow model is used which retains the essential features of the actual mixing process, the resulting system of equations is amenable to manual calculation. The theory is correlated with data obtained from an annular nozzle wind tunnel having a central supersonic core of rocket exhaust gases. Correlation is also made with</p>  | <ol style="list-style-type: none"> 1. Supersonic diffusers 2. Turbulent flow 3. Compressible flow 4. Jet mixing flow 5. Theory 6. Tests I. AFSC Program Area 750G, Project 6850, Task 685002 II. Contract AF 40(600)-800 S/A 24(61-73) III. ARO, Inc., Arnold AF Sta, Tenn. IV. Peters, C. E. and Wehofer, S. V Available from OTS VI In ASTIA collection | <p>Arnold Engineering Development Center Arnold Air Force Station, Tennessee</p> <p>Rpt. No. AFDC-TR-61-18. CONSTANT AREA MIXING OF NON-ISOENERGETIC COAXIAL COMPRESSIBLE STREAMS. January 1962, 43 p. incl 9 refs., illus., table. Unclassified Report</p> <p>As part of a general investigation of two-stream supersonic diffusers, a theory was developed for the bounded turbulent mixing between coaxial high velocity streams of different composition and temperature. The usual assumption of negligible axial pressure gradient is invalid in this case because the outer stream is bounded by a cylindrical duct. A highly simplified flow model is used which retains the essential features of the actual mixing process; the resulting system of equations is amenable to manual calculation. The theory is correlated with data obtained from an annular nozzle wind tunnel having a central supersonic core of rocket exhaust gases. Correlation is also made with</p>  | <ol style="list-style-type: none"> 1. Supersonic diffusers 2. Turbulent flow 3. Compressible flow 4. Jet mixing flow 5. Theory 6. Tests I. AFSC Program Area 750G, Project 6850, Task 685002 II. Contract AF 40(600)-800 S/A 24(61-73) III. ARO, Inc., Arnold AF Sta, Tenn. IV. Peters, C. E. and Wehofer, S. V Available from OTS VI. In ASTIA collection |
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